Forecast and Backcast of the Solar Cycles

K. M. HIREMATH
Indian Institute of Astrophysics, Bengaluru-560034, India
E-mail: hiremath@iiap.res.in

Abstract

Solar cycle is modeled as a forced and damped harmonic oscillator and the amplitudes, frequencies, phases and decay factors of such a harmonic oscillator are estimated by non-linear fitting the equation of sinusoidal and transient parts to the sunspot and irradiance (proxy for the sunspot) data for the years 1700-2008. We find that:(i) amplitude and frequency (or period of ∼11 yr) of the sinusoidal part remain constant for all the solar cycles; (ii) the amplitude of the transient part is phase locked with the phase of the sinusoidal part; (iii) for all the cycles, the period and decay factor (that is much less than 1) of the transient part remain approximately constant. The constancy of the amplitudes and the frequencies of the sinusoidal part and a very small decay factor from the transient part suggests that the solar activity cycle mainly consists of a persistent oscillatory part that might be compatible with long-period (∼22 yr) Alfven oscillations. For all the cycles, with the estimated physical parameters (amplitudes, phases and periods) and, by an autoregressive model, we forecast (especially for coming solar cycle 25) and backcast (to check whether Maunder minimum type solar activity exists or not) the solar cycles. We find that amplitude of coming solar cycle 25 is almost same as the amplitude of the previous solar cycle 24. We also find that sun might not have experienced a deep Maunder minimum (MM) type of activity during 1645-1700 AD corroborating some of the paleoclimatic inferences and, MM type of activity will not be imminent in near future, until at least 200 years.

1 Introduction

Owing to proximity of the sun, we not only receive the light for sustenance of flora and fauna's life, but also cyclic and sporadic sun's activities that havoc the electrical grids, satellite communications and life of the satellites.

In addition to sustenance of life on this Earth, our nearest star also influences the climate and environment of the Earth. Recent overwhelming evidences are building up that sun indeed influences the earth's climate, especially Indian Monsoon rainfall (Hiremath and Mandi 2004; Hiremath 2009; Hiremath, Hegde and Soon 2015 and references there in). Considering these important facts, it is essential to understand the origin of solar cycle and activity phenomena and magnitude of their prediction in future. Although our understanding of origin of solar cycle and activity phenomena is far from reality, by learning from the variability of sun's long term observed sunspot activity over a century scale is very useful and most reliable in projecting the future activity.

Present study aims at this direction and long term variation of sunspot activity is described as a forced and damped harmonic oscillator. Solution of such a forced and damped harmonic oscillator is subjected to a non-liner least square fit to the long term sunspot activity data and, amplitudes, frequencies and phases of long period ($\sim$ 22 yr) oscillations are obtained. With these physical parameters and by the method autoregression, sunspot activity after cycle 24 and before (cycle 1) the era of regular observations are obtained. In the previous study (Hiremath 2006; Hiremath 2008), we have done the similar analysis. However, present study differs in two aspects: (i) back cast of the solar cycles before the era of so called 1st cycle is obtained and, (ii) recently updated sunspot data as compiled by "Royal Observatory of Belgium (http://www.sidc.be/silso/)" is used for prediction of future cycles 24 and beyond.

Plan of the presentation is as follows. In section 2, data and method of analyses are presented. Section 3 describes the results and, brief conclusions are presented in section 4.

2 Data and Analysis

For the years 1755-2008, we consider the recent version of monthly sunspot number data as compiled by "WDC-SILSO, Royal Observatory of Belgium, Brussels" (http://www.sidc.be/silso/). Before the year 1755, only yearly Belgium data is available. For this purpose, in order to keep the uniformity and as described in the following, this yearly data is not suitable for getting the very good fit from equation (1). Hence, for back cast of activity of solar cycles, before the year 1755 AD, for the years 1700-1755, modeled total

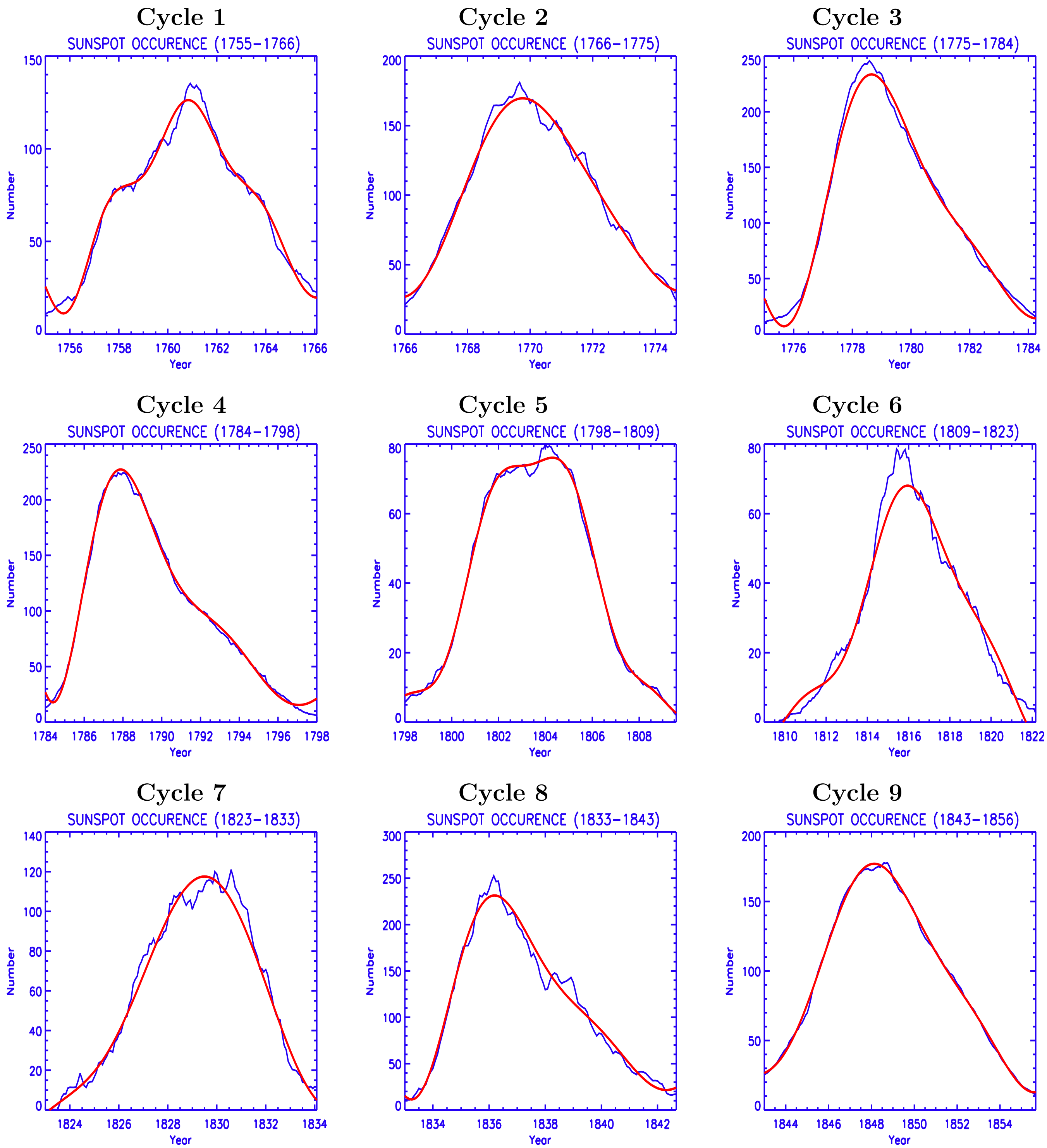


Figure 1: For the solar cycles 1-9, nonlinear least square fit of a solution of forced and damped harmonic oscillator. Blue continuous line is the observed sunspot data and red continuous line is obtained from the fit.

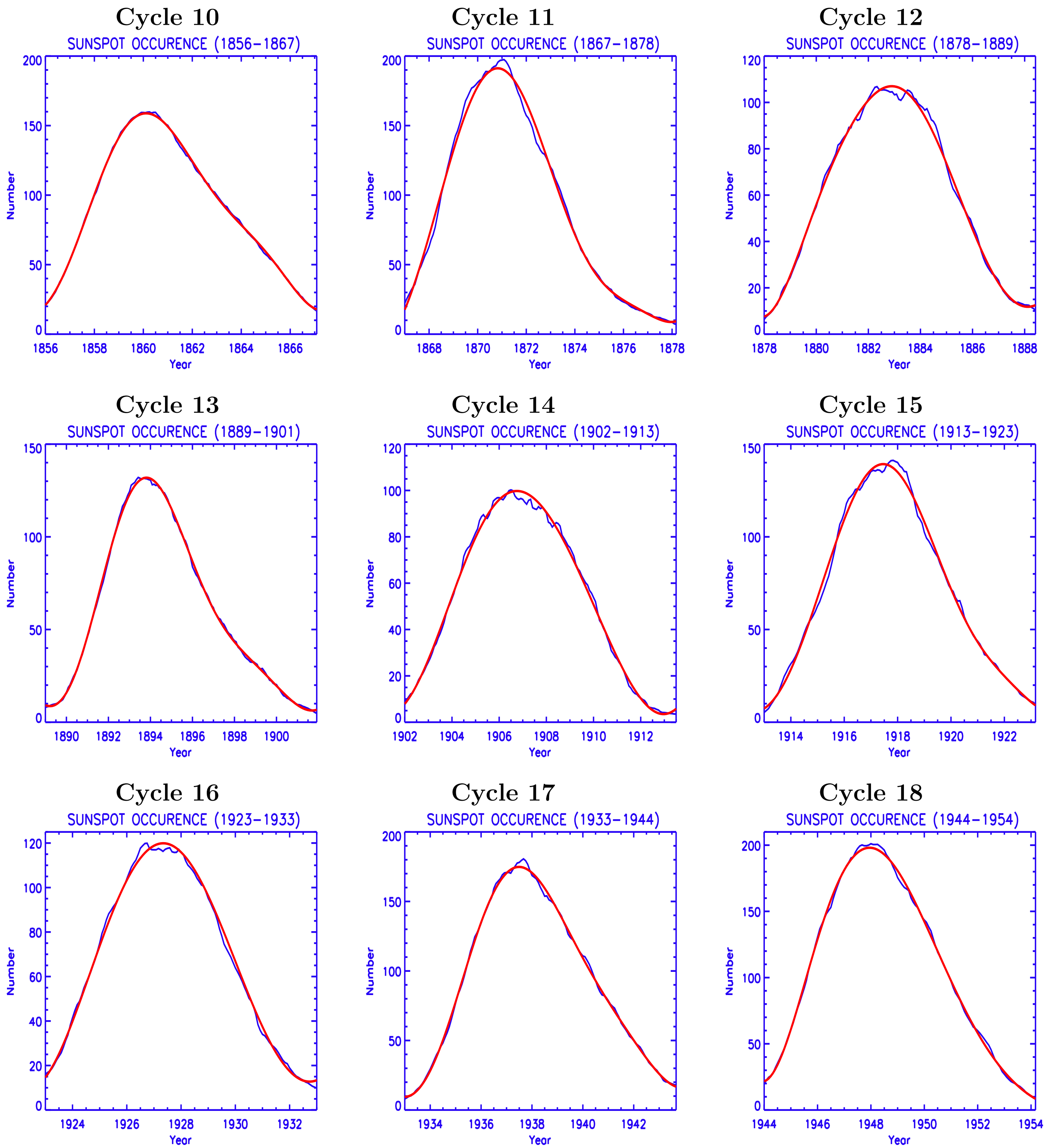


Figure 2: For the solar cycles 10-18, nonlinear least square fit of a solution of forced and damped harmonic oscillator. Blue continuous line is the observed sunspot data and red continuous line is obtained from the fit.

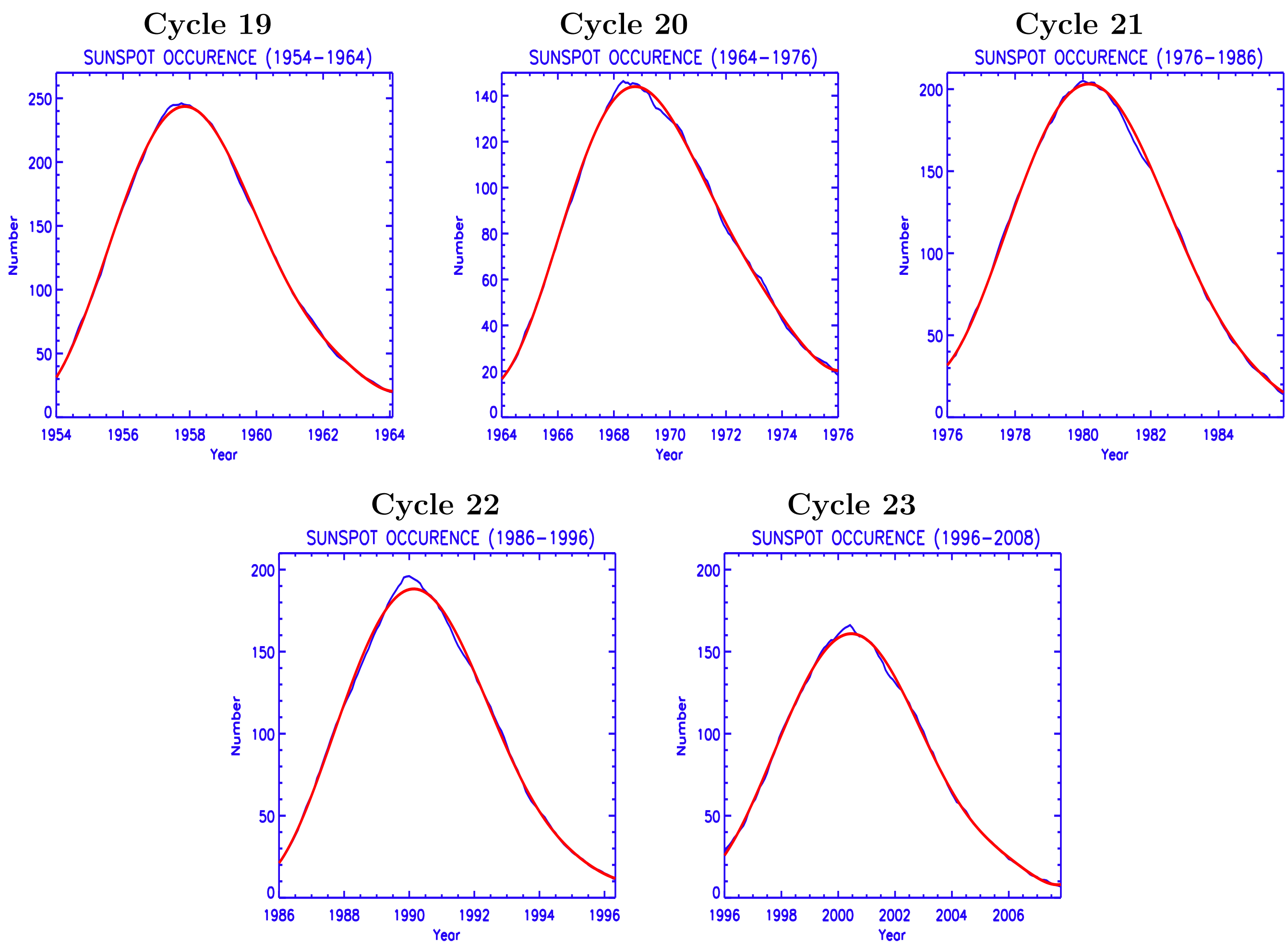


Figure 3: For the solar cycles 19-23, nonlinear least square fit of a solution of forced and damped harmonic oscillator. Blue continuous line is the observed sunspot data and red continuous line is obtained from the fit.

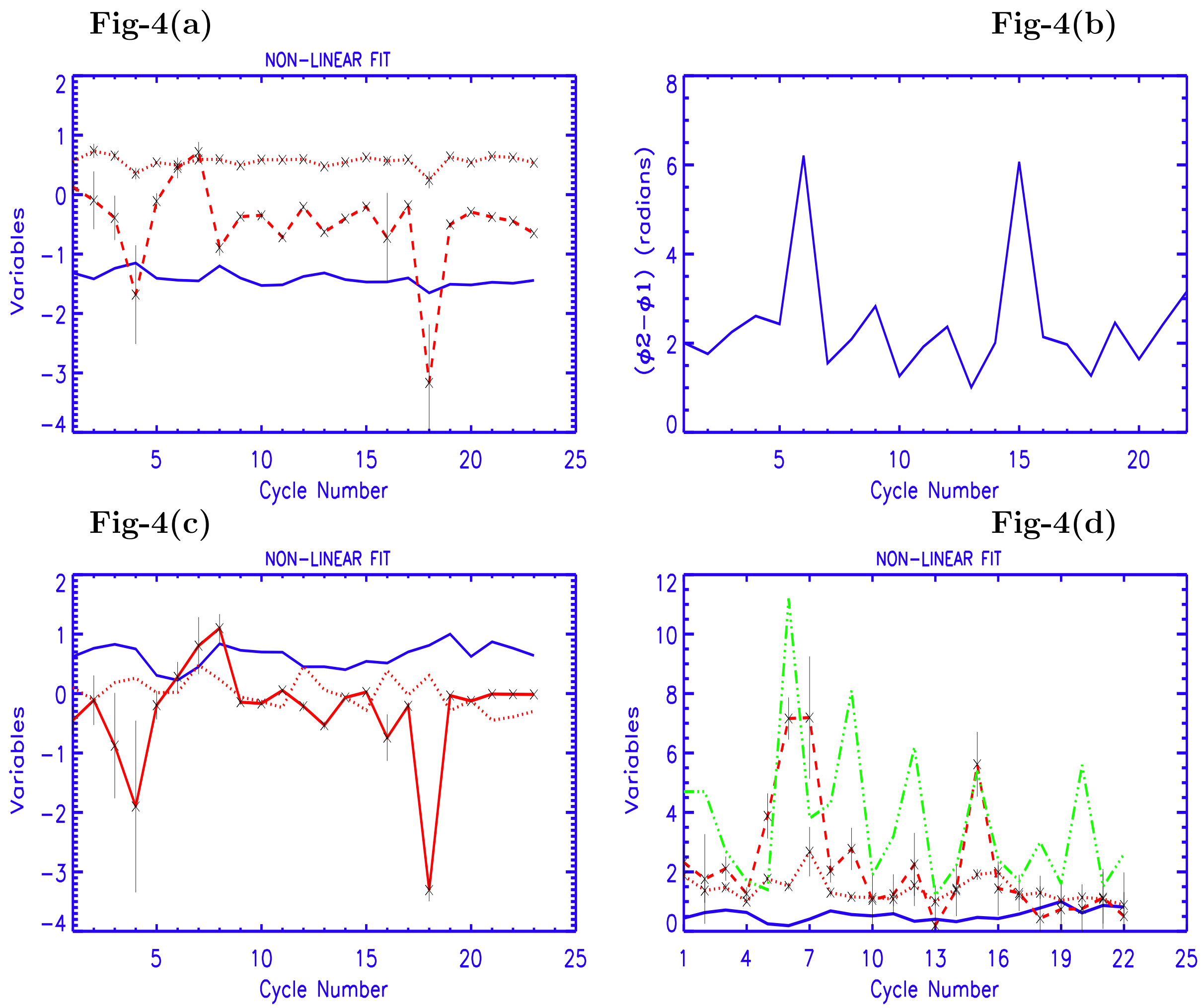


Figure 4: For the solar cycles 1-23, Fig 4(a) illustrates the variation of different coefficients (blue continuous line is the amplitude A_1, the dotted line is the frequency ω and the dashed line is the phase ϕ_1) for the steady part. Whereas variation of different physical parameters of transient part are illustrated in Fig 4(c) and Fig 4(d) respectively. In Fig 4(c), red continuous line is the amplitude A_2 and the dotted line represents the decay factor γ respectively. In Fig 4(d), the dotted line is the frequency ω' and the dashed line is the phase ϕ_2. The dash with three dotted line represents the values of χ^2 (a measure of goodness of fit) for each cycle. Fig 4(b) illustrates the phase difference between steady and transient parts. In both the illustrations of Fig 4(c) and Fig 4(d) blue continuous line is the cycle mean (normalized to the maximum number in all the solar cycles).

irradinace (kindly provided by Dr. Lean, Naval Observatory, USA; Lean and Brueckner 1989; Lean 1990; Lean 1991) data is used for reconstruction of the sunspot data.

Following the previous study (Hiremath 2006), monthly sunspot data is normalized as follows. If x_i are monthly means of sunspot number, $\bar{x}$ is the cycle mean and σ is the standard deviation, then the normalized deviation of sunspot data is $y_i = (x_i - \bar{x})/\sigma$. With the Levenberg-Marquardt algorithm (Press *et.al* 1992), such a normalized sunspot data is subjected to non-linear least square fit for the following solution of a forced and damped harmonic oscillator

$$y = A_1 \cos(\omega t - \phi_1) + A_2 \cos(\omega' t - \phi_2) \mathrm{e}^{-\gamma t}, \tag{1}$$

where y is displacement (in the present context we consider sunspot number), A_1, A_2 are amplitudes, $\omega(2\pi/T$, where T is the period in years) is the sinusoidal frequency, ω' $(2\pi/T'$, where T' is the period in years) is the damping frequency, ϕ_1 and ϕ_2 are the phases, γ is the decay factor and t is the time variable in months. On RHS of above equation, first term is called the "steady" and second term the "transient" parts of solution of a forced and damped harmonic oscillator (Tipler and Mosca 2003). For different solar cycles from 1-23, observed sunspot data (blue curve) with the over plotted non-linear least square fit (red continuous line) is illustrated in Figures 1-3 respectively.

3 Results

3.1 Steady and Transient parts During Cycles 1-23

For different cycles 1-23, Figure 4 illustrates the variation of different physical parameters of the steady and transient parts of solution of a forced and damped harmonic oscillator. For the steady part, Fig 4(a) illustrates cycle to cycle variation of amplitude A_1 (blue continuous line), frequency ω (red dotted line) and the phase ϕ_1 (red dashed line). Whereas physical parameters of transient part such as amplitude A_2 (red continuous line) and decay factor γ (dotted line) are presented in Fig 4(c). Figure 4(d) illustrates frequency $\omega^{'}$ (red dotted line) and phase ϕ_2 (red dashed line). In both the figures (4(c) and 4(d)), blue continuous line represents the cycle mean (normalized to the maximum value in all the solar cycles).

As in the previous study (Hiremath 2006), one can notice from three Figures (4(a), 4(c) and 4(d)) that frequency ($\sim$ 22 yrs), amplitude and phase (except during few years) of the steady part of the solar oscillator remains almost constant. Hence, an inevitable conclusion, as in the previous study (Hiremath 2006), is that there is a constant perturber in the deep interior that creates and maintains the solar cycle and activity phenomena with near periodicity of 22 yrs.

In the previous (Hiremath 2006) study, we find that whenever difference in phase of steady and transient parts reaches a maximum, solar cycle and activity phenomena on longer time scales ($\sim$ 100 yrs) attains a deep minimum, although not as deep as so called Maunder minimum type of activity. For both the data set, in Fig 4(b), phase difference ($\phi_1 - \phi_2$) between the steady and transient parts is illustrated. One can notice that nearly for every 100 years, sunspot activity is minimum.

3.2 Activity Before the Era of Cycles 1-23

Let us examine whether all the physical parameters of steady part of solution of a forced and damped harmonic oscillator are constant or not during the 16th century. Another investigation is to confirm whether sun really experienced a Maunder type of deep minimum or not. As there are no systematic sunspot observations before the era of cycle 1, we follow the following method in order to reconstruct the sunspot data. For the years 1700-1755, Lean's total irradiance data is considered, and different cycles minima are estimated. Before the era of regular sunspot data that starts with cycle 1, first previous cycle data is assigned as -1 cycle, next cycle -2 and so on.

For the period of 1755-2008, sunspot data from the regular observations is considered and association is examined with the Lean's reconstructed TSI data for the same period. We find that both the variables are strongly positively correlated and the estimated correlation is 96%. Such an association and hence a scatter plot between two variables is illustrated in Fig 5(a). Both the variables are also subjected to a linear least square fit and the following law between the two variables is obtained

$$S = [-(2.188 \pm 0.131) + (0.016 \pm 0.000096)I]10^5, \qquad (2)$$

where S is the sunspot activity and I is reconstructed total radiance. With this empirical relation, sunspot data is reconstructed during 1700-1750. As

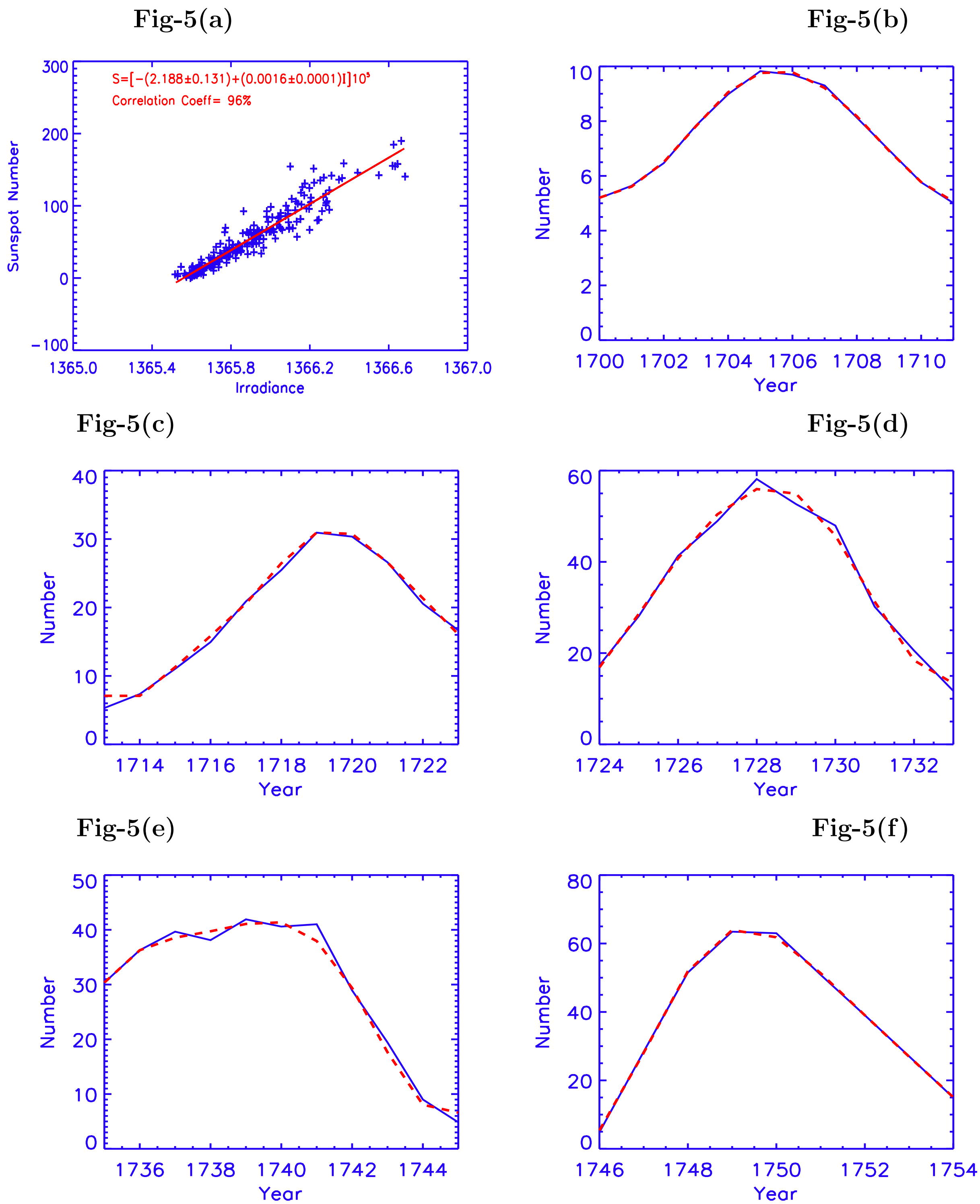


Figure 5: Fig 5(a) illustrates a scatter plot between the total irradiance and the sunspot number. Overplotted red continuous line represents the least square fit between two variables. Whereas, for the cycles -5 to -1, Fig-5(b) to Fig-5(f) represent the monthly mean variation of Lean's total irradiance (blue continuous line) data overplotted (dashed red line) with the reconstructed sunspot data.

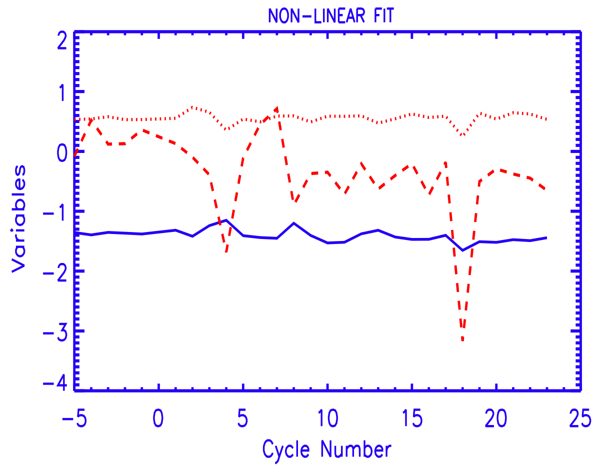
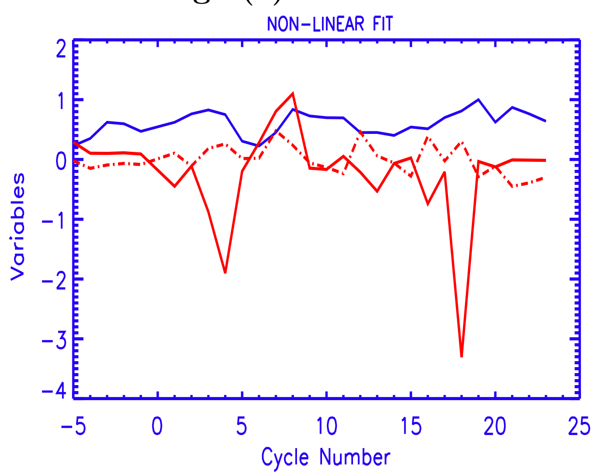


Fig-6(a) **Fig-6(b)**



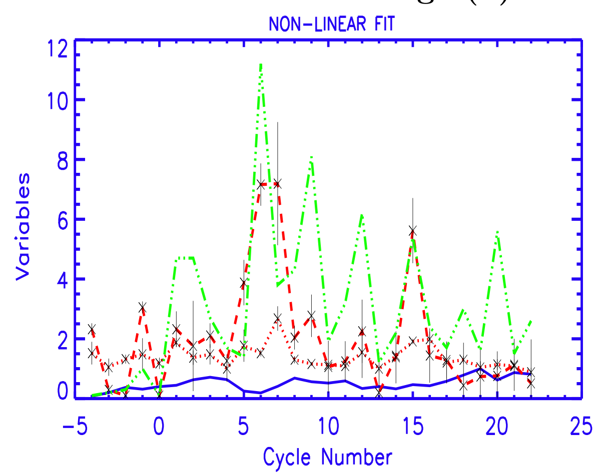


Figure 6: For the extended cycles (-5 to 23), a nonlinear least square fit of a solution of forced and damped harmonic oscillator. For the extended solar cycles -5 to 23, upper panel of Fig 6 illustrates the variation of different coefficients (blue continuous line is the amplitude A_1, the dotted line is the frequency ω and the dashed line is the phase ϕ_1) for the steady part. Whereas variation of different physical parameters of transient part is illustrated in Fig 6(a) and Fig 6(b) respectively. In Fig 6(a), red continuous line is the amplitude A_2 and the dash with dotted line represents the decay factor γ respectively. In Fig 6(b), the dotted line is the frequency ω' and the dashed line is the phase ϕ_2. The dash with three dotted line represents the values of χ^2 (a measure of goodness of fit) for each cycle. For all the cycles -5 to 23, in both the figures Fig-6(a) and Fig-6(b), blue continuous line is cycle mean of sunspot number.

described in the previous section, with a solution of forced and damped harmonic oscillator, such a reconstructed sunspot activity is subjected to a nonlinear least square fit and different physical parameters are estimated For the cycles -1, -2, -3, -4, -5, Figures 5(b)-5(f) illustrate the reconstructed sunspot activity (blue continuous line) with over plotted fit (red dashed line). Whereas Fig 6 represents different physical parameters of steady and transient parts of a forced and damped harmonic oscillator. One can notice that, as in section 3.1, for all the solar cycles (-5 to 23), frequency ($\sim$ 22 yrs) and amplitude of the steady part remains approximately constant and amplitude of the transient part is phase locked with the steady part.

3.3 Forecast of the Solar Cycles

There are many studies (Quassim, Attia and Elminir 2007; Bhatt, Jain and Aggarwal 2009; Uwamacharo and Cilliers 2009; Volobuov 2009; Kilcik *et.al.* 2009; Kane 2010, Baolin 2011; Kakad 2011; Nobel and Whatland 2012; Du 2012; Pesnell 2012; Du 2012; Yu et.al. 2012; Uzal, Piacentini and Verdes 2012; Attia and Hassan 2013; Kane 2013; Helal and Gelal 2013; Pishkalo 2014; Kilcik *et.al.* 2014; Li, Feng and Li 2015; Janardhan *et.al.* 2015; Raschna and Sarychov 2015; Gao 2016; Hathaway and Upton 2016; Deminov, Nepomnyaschaya and Obridko 2016; Kitashivili 2016; Travaglini 2017; Gopalswamy *et.al.* 2018; Pesnell 2018; Pesnell and Schatten 2018; Sabarinath and Anilkumar 2018; Okoh *et.al.* 2018; Petrovay *et.al.* 2018; Singh and Bhargawa 2019; Pala and Atici 2019; Okoh and Okoro 2020; Miao *et.al.* 2020; Wu and Qin 2021; Velasco *et.al.* 2021; Maddanuand and Proietti 2022; Du 2022; Dang *et.al.* 2022) that forecast amplitudes of cycles 24 and 25. There are also excellent reviews (Petrovay 2010; Pesnell 2012; Lopes *et.al.* 2014; Tripathi 2016; Petrovay 2020; Nandy 2021) on the prediction of solar cycle. Whereas forecast of solar cycles beyond 24 and 25 are rare. In addition, forecast of cycle periods is at most importance for the future space missions and space weather predictions.

Using sunspot data and autoregressive model, for the years of 1755-1996, in the previous study (Hiremath 2006), physical parameters of steady part of forced and damped harmonic were used to forecast the amplitudes and periods of steady part. In this study, as we have extended sunspot data set from 1700-2008, whole of this data set is used for estimation of different physical parameters of steady and transient parts. With these extended physical parameters and the autoregression model (see the description as given in section

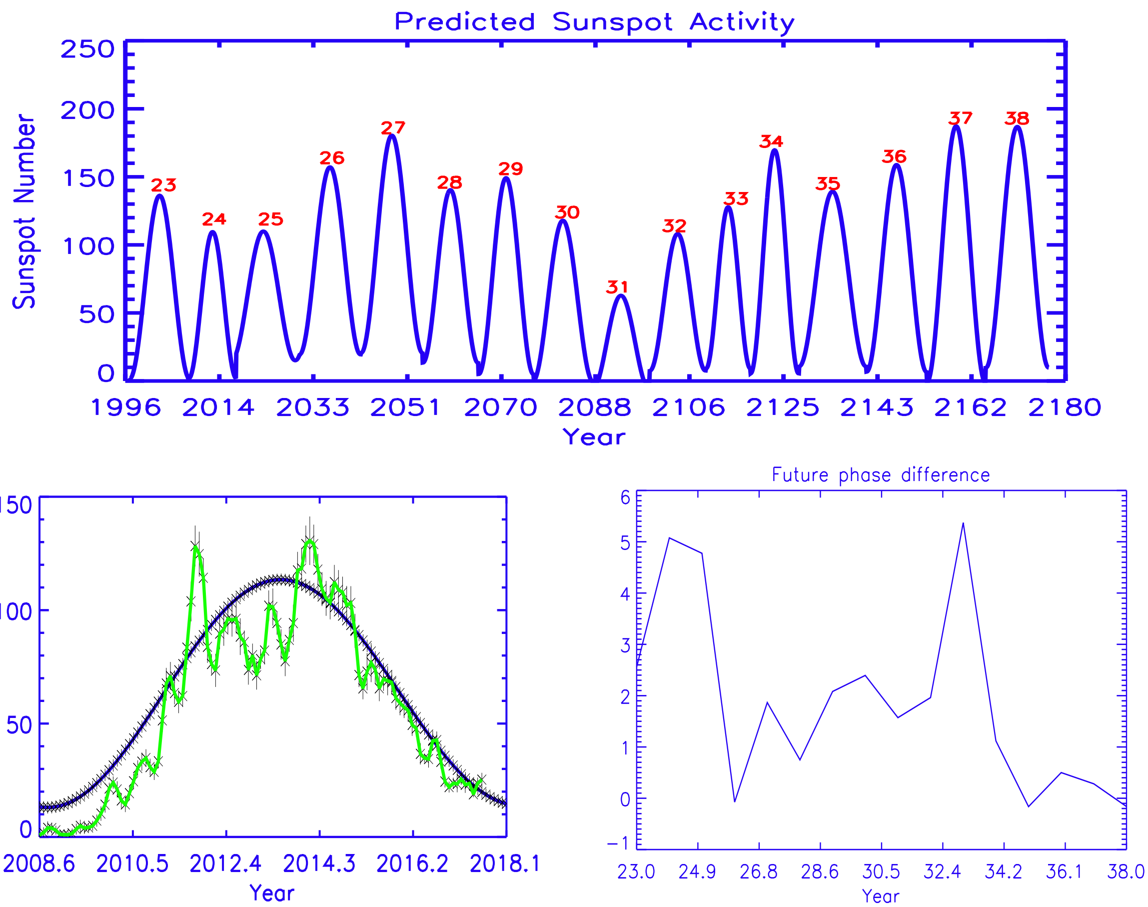


Figure 7: For the solar cycles 23 and beyond forecast of amplitudes of future 16 solar cycles (upper panel). For typical cycle 24, first figure of lower panel illustrates the monthly variation of predicted (blue continuous line with error bars) and the observed (green continuous line with error bars) sunspot data. Where as second figure in the lower panel illustrates the predicted phase difference between the steady and transient parts for the future cycles.

Fig-8(a) **Fig-8(b)**

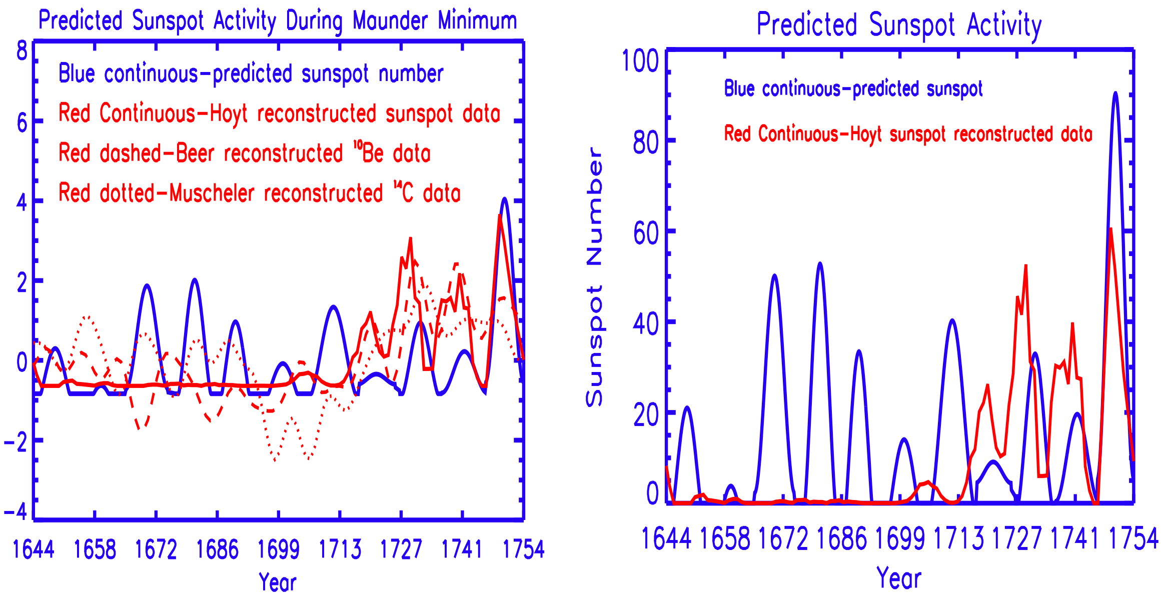


Figure 8: For the solar cycles -1 to -5 and beyond upto 1640 AD, backcast of the solar activity. Fig 8(a) illustrates the deviation (from the mean and normalized with their respective standard deviations) of different solar and plaeoclimatic activities. Blue continuous line is sunspot activity obtained from the backcast. Red continuous line is Hoyt's reconstructed sunspot data. Red dashed line is reconstructed $Be10$ data from Beer *et.al.* (1998) and, red dotted line is reconstructed $C14$ data from Muscheler *et.al.* (2007). For the same cycles, whereas Fig 8(b) illustrates the absolute (without normalization) sunspot (blue continuous line) data from backcast and, Hoyt reconstructed sunspot data (red continuous line).

1, page 46, second column, Hiremath 2008), physical parameters of steady and transient parts are estimated for the 16 cycles (cycles 23-38). With the predicted physical parameters, monthly sunspot data is reconstructed from the physical parameters of steady part as this term mainly reproduces the amplitude and period of a solar cycle, Top panel of Fig 7 illustrates the temporal variation of amplitudes of solar cycles 23 and beyond. For the sake of comparison with the observed sunspot data, for typical solar cycle 24, first figure of lower panel of Fig 7 illustrates the predicted (blue continuous line with error bars) amplitude of the solar cycle. We find the stunning result that not only the amplitude but also the period of the predicted cycle 24 nearly matches very well with the observed amplitudes and period of the solar cycle.

Another interesting part of our study is that whenever the difference between the phases of steady and transient parts attain the maximum, for every 100 years, few cycles around that period have minimum sunspot activity, although it is not as deep minimum as so called Maunder minimum type activity. For example (see upper and lower panels of Fig 7) around 2100 AD, amplitude of sun's activity probably reaches very low and minimum. However, notice that the predicted sunspot activity beyond cycle 24, as argued by some studies (Janardhan *et.al.* 2015; Sancez-Sesma 2016; Zharkova *et.al.* 2015), will never attains a Maunder minimum type of solar activity at least upto 200 yrs.

3.4 Backcast of the Solar Cycles

Using same physical parameters of steady part of forced and damped harmonic oscillator for the years 1700-2008, with the autoregressive model, we backcast the amplitude of the solar cycle upto the years 1645. Idea of this exercise is to examine whether sun really experienced a deep Maunder minimum (during the period of 1645-1700 AD) type of solar activity. Figure 8 illustrates the backcasted solar activity. Fig 8(a) illustrates the deviation (from the mean and normalized with their respective standard deviations) of different solar and plaeoclimatic activities. Blue continuous line is sunspot activity obtained from the backcast. Red continuous line is Hoyt's reconstructed sunspot data. Red dashed line is reconstructed $Be10$ data from Beer *et.al.* (1998) and, red dotted line is reconstructed $C14$ data from Muscheler *et.al.* (2007). For the same cycles, whereas Fig 8(b) illustrates the absolute (without normalization) sunspot (blue continuous line) data from backcast and, Hoyt reconstructed sunspot data (red continuous line). Of course, one can notice from these figures that, although we find a dearth of solar activity around 1650-1662, our backcast sunspot data did not reproduce so called deep Maunder minimum of solar activity during 1600-1645. Infact, these results are also consistent with the previous (Komitov and Kaftan 2003; Hiremath 2010; De Jager and Duhau 2012; Steinhilber and Beer 2013; Velasco Herrara *et. al.* 2015; Gao 2016; Svalgaard and Schatten 2016; Travglini 2017) studies which do not agree with the idea that there was a dearth of sunspot activity in particular and sun's other activity (such as coronal holes, coronal mass ejections, flares etc.,) in general (see Feyman 1982; Legrand and Simon 1991; Cliver, Boriakoff and Bounar 1998; Feyman 1982; De Jager 2005; Georgieva and Kirov 2006) around 1645-1700 AD as claimed by Maunder. However, further analysis is required to confirm this result as linear autoregressive method has certain limitations.

4 Conclusions

Shape of the solar cycle is described as a solution of a forced and damped harmonic oscillator. First, for the years 1700-2008 (cycles 1-23), updated monthly sunspot data is subjected to a nonlinear least square fit of this solution and different physical parameters (amplitudes, phases and frequencies) of steady and transient parts of the solar oscillator are estimated. Following are the important findings: (i) for all the cycles -5 to -1 and 1-23, amplitude, frequency (or period of $\sim$ 22 yrs) and the dissipation factor γ remain approximately constant, (ii) amplitude of the transient part is phase locked with the phase of the steady part, (iii) whenever phase difference between the steady and transient parts attains a maximum, sunspot activity also reaches a minimum, although this minimum is not as deep as so called Maunder minimum and, (iv) for the period 1755-2008, we find a linear relationship (with a high correlation) between the sunspot and total irradiance activities.

Before the era of regular observations of sunspots, from 1700-1755, Lean's total irradiance data is considered and, by detecting different minima, 5 cycles (-1 to -5) are obtained. From the obtained relationship between sunspot and total irradiance data, sunspot number data is reconstructed for all the 5 cycles (1700-1755). All these five cycles sunspot data is subjected to a nonlinear least square fit of a forced and damped harmonic oscillator and, different physical parameters of steady and transient parts are estimated.

We find that cycle to cycle variation of all the physical parameters estimated from these 5 cycles data is almost same as cycle to cycle variation of physical parameters estimated from 1-23 cycles.

These 5 cycles physical parameters are merged with the physical parameters of cycles 1-23 estimated from the systematic sunspot observations. Finally, with an autoregressive model, different physical parameters that are estimated from the combined data set are used for forecast and back cast of the amplitude and periods of the solar cycles. Important results are: (i) predicted amplitude and period of cycle 24 are almost same as the amplitude and period of the observed sunspot cycle, (ii) amplitude of upcoming solar cycle 25 is almost same as present 24th cycle, (iii) as expected by other studies, there is no imminent so called deep Maunder minimum type of solar activity in future, at least upto 200 yrs and, (iv) backcasted sunspot activity suggests that although there are intermittent dearth of sunspot activity, we did not find a very long (1645-1700) deep minimum of solar activity as claimed by Maunder.

References

[1] Alfven, H., Arkiv Math Astr Fys, **29A**, 1943, p. 12

[2] Attia, A, Ismail, H. A, Basurah, H. M., Astrophys and Space Sci, **344**, 2013, p. 5

[3] Baolin, T, Astrophys and Space Sci, **332**, 2011, p. 65

[4] Bhatt, N. J., Jain, R, Aggarwal, M., Solar phsyics, **260**, 2009, 225

[5] Beer, J., Tobias and Weiss, W., Sol Phys., **181**, 1998, p. 237

[6] Cliver E W Boriakoff V and Feyman J, Geophys Res Let **25**, 1998, p. 1035

[7] De Jager C, Space Sci Rev, **120**, 2005, p. 197

[8] De Jager, C and Duhau, S, Journal of Space Weather and Space Climate, **2**, 2012, p. A07

[9] Dicke, R. H, Nature, **276**, 1978, p. 676

[10] Deminov, M. G., Nepomnyashchaya, E. V, Obridko, V. N, Ge&Ae, **56**, 2016, p. 742

[11] Feynman J, JGR, **87**, 1982, p. 6153

[12] Dang, Yuchen, Chen, Ziqi, Li, Heng and Shu, Hai, eprint arXiv:2203.05757, 2022

[13] Du, Z. L, Astrophys and Space Sci, **338**, 2012, p. 9

[14] Du, Z. L, Astrophysics and Space Science, **367**, 20220, p. 20

[15] Georgieva K and Kirov B, Sun and Geosphere, **1**,2006 , p. 12

[16] Gao, P. X, Astrophysical Journal, **830**, 2016, p. 140

[17] Gopalswamy, N, Mäkelä, P, Yashiro, S and Akiyama, S, Journal of Atmospheric and Solar-Terrestrial Physics, **176**, 2018, p. 26

[18] Hathaway, D. H and Upton, L. A, JGRA, **121**, 2016, p. 10744

[19] Hathaway, D. H, Living Reviews in Solar Physics, **12**, 2015, p. 87

[20] Hiremath, K. M, Ph.D thesis, Bengaluru Universitiy, 1994
see also http://hdl.handle.net/2248/122

[21] Hiremath, K. M and Gokhale, M. H, Astrophysical Journal, **448**, 1995, p. 437

[22] Hiremath, K. M. and Mandi, P. I, New Astronomy, **9**, 2004, p. 651

[23] Hiremath, K. M, Sun and Geo, **4**, 2009, p. 16

[24] Hiremath, K. M, Sun and Geosphere, **5**, 2010, p. 17

[25] Hiremath, K. M, Astronomy and Astrophysics, **452**, 2006, p. 591

[26] Hiremath, K. M, Astrophysics and Space Science, **314**, 2008, p. 45

[27] Hiremath, K. M., Manjunath, Hegde and Soon, W, New Astronomy, **35**, 2015, p. 8

[28] Helal, H. R and Galal, A. A, Journal of Advance Research, **4**, 2013, p. 275

[29] Janardhan, P, Bisoi, S. K, Ananthakrishnan, S, Sridharan, R and Jose, L, Sun and Geosphere, **10**, 2015, p.147

[30] Janardhan, P, Bisoi, S K, Ananthakrishnan, S, Tokumaru, M, Fujiki, K, Jose, L and Sridharan, R, JGRA, **120**, 2015, p. 5306

[31] Kakad, B, Solar Physics, **270**, 2011, p. 393

[32] Kane, R. P, AnGeo, **28**, 2010, p. 1463

[33] Kane, R. P, Solar Physics, **282**, 2013, p. 87

[34] Kitiashvili, Irina N, Astrophysical Journal, **831**, 2016, 15

[35] Kilcik, A., Anderson, C. N. K., Rozelot, J. P., Ye, H., Sugihara, G and Ozguc, A, Astrophysical Journal, **693**, 2009, p. 1173

[36] Kilcik, A, Yurchyshyn, V. B, Ozguc, A, Rozelot, J. P, Astrophysical Journal, **794**, 2014, p. 2

[37] Komitov, B. P.; Kaftan, V. I, Ge&Ae, **43**, 2003, p. 553

[38] Lean, J. L and Brueckner, G. E., Astrophysical Journal, **337**, 1989, p. 568

[39] Lean, J. L, ApJ, **363**, 1990, p. 718

[40] Lean, J. L, 1991, Rev. Geophys., **29**, No 4, 1991, p. 505

[41] Li, K. J., Feng, W and Li, F. Y, JASTP, **135**, 2015, p. 72

[42] Legrand J P and Simon P A, Solar Physics, **131**, 1991, p. 187

[43] Lopes, I, Passos, D, Nagy, M and Petrovay, K, Space Sci Rev, **186**, 2014, p. 535

[44] Maddanu, Federico and Proietti, Tommaso, Solar Physics, *297*, 2022, p. 13

[45] Miao, Juan, Wang, Xin, Ren, Ting-Ling and Li, Zhi-Tao, Research in Astronomy and Astrophysics, **20**, 2020, p. 8

[46] Muscheler, R ., et. al, Queternary Science Reviews, **26**, 2007, p. 82

[47] Nandy, Dibyendu, Solar Physics, **296**, 2021, p. 54

[48] Noble, P. L and Wheatland, M. S, Solar Physics, **276**, 2012, p. 363

[49] Okoh, D. I, Seemala, G. K, Rabiu, A. B, Uwamahoro, J, Habarulema, J. B and Aggarwal, M, Space Weather, Volume 16, Issue 9, 2018, pp. 1424-1436

[50] Okoh, Daniel and Okoro, Eucharia, Solar Physics, **295**, 2020, p. 1

[51] Pala, Zeydin and Atici, Ramazan, Solar Physics, **294**, 2019, p. 14

[52] Pesnell, W. Dean, Space Weather, **16**, 2018, pp 1997

[53] Pesnell, W. Dean and Schatten, Kenneth H, Solar Physics, **293**, Issue 7, 2018, p. 10

[54] Petrovay, K. 2010, Living Review of Solar Physics, **7**, 2010, p. 6

[55] Petrovay, K, Nagy, M, Gerják, T and Juhász, L, Journal of Atmospheric and Solar-Terrestrial Physics, **176**, 2018, p. 15

[56] Petrovay, K, 2020, Living Reviews in Solar Physics, **17**, 2020, 96 pages

[57] Pesnell, W. D, Solar Physics, **281**, 2012, p. 507

[58] Pishkalo, M. I, Solar Physics, **289**, 2014, p. 1815

[59] Press, W. H, Teukolsky, S. A, Vetterling, W. T, Flannery, B. P, in Numerical Recipes in C (Cambridge University Press), second edition, 1992, p. 640

[60] Quassim, M, S, Attia, A, F and Elminir, H. K, Solar Physics, **243**, 2007, p. 253

[61] Roshchina, E. M and Sarychev, A. P, Ge&Ae, **55**, 2015, p. 892

[62] Sabarinath, A and Anilkumar, A. K, Journal of Earth System Science, **127**, 2018, p. 15

[63] Sánchez-Sesma, J, Earth Syst. Dynam., **7**, 2016, p. 583

[64] Singh, A. K. and Bhargawa, Asheesh, Astrophysics and Space Science, **364**, 2019, p. 7

[65] Steinhilber, F and Beer, J, JGRA, **118**, 2013, p. 1861

[66] Svalgaard, L and Schatten, K. H, Solar Physics, **291**, 2016, p. 2653

[67] Tipler, P. A and Mosca, G, in Physics for Scientists and Engineers (New York: Freeman and company), 2003, p. 451

[68] Travaglini, G, Solar Physics, **292**, 2017, p. 23

[69] Tripathy, S. C, Asian Journal of Physics, **25**, 2016, p. 387

[70] Uwamahoro, J, McKinnell, L and Cilliers, P. J, JASTP, **71**, 2009, p. 569

[71] Uzal, L. C, Piacentini, R. D, Verdes, P. F, Solar Physics, **279**, 2012, p. 551

[72] Velasco Herrera, V. M., Mendoza, B and Velasco Herrera, G, New Astronomy, **34**, 2015, p. 221

[73] Velasco Herrera, V. M.; Soon, W. and Legates, D. R, Advances in Space Research, **68**, 2021, p. 1485

[74] Volobuev, D. M, Solar Physics, **258**, 2009, p. 319

[75] Waldmeier, M. 1961, in The Sunspot-Activity in the Years 1610-1960, 1961, 24

[76] Wu, S. -S and Qin, G., arXiv:2102.06001, 2021

[77] Yu, Y, van Dyk, D. A, Kashyap, V. L, Young, C. A, Solar Physics, **281**, 2012, p. 847

[78] Zharkova, V. V, Shepherd, S. J, Popova, E and Zharkov, S. I, Nature Scientific Reports, **5**, 2015, p. 15689